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Compression Buckling Response of Tailored Rectangular Composite Plates

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Buckling resistance is often a controlling criterion in the design of structures composed of plate elements. Design concepts that lead to increased buckling loads (or strains) of these plate elements can directly lower the structural cost and/or weight by a number of means. This study quantifies the improvements that can be achieved in compression buckling loads of rectangular composite plates by using a simple stiffness-tailoring concept. The approach is to position the unidirectional lamina through the thickness and over the planform of the plate so that the buckling load is increased with no loss in in-plane stiffness or increase in weight. Finite element analyses have been used to determine the effects of tailoring on the buckling loads of plates with various boundary conditions, aspect ratios, thicknesses, and membrane stiffnesses. Increases in buckling loads (or strains) of 200% or more compared to the uniform plate-buckling loads are shown possible with this tailoring concept.

Introduction

THE design of aircraft primary structural elements is often strongly influenced by a plate-buckling criterion. For example, the wing cover surface is normally restricted from buckling at loads up to maximum operating or even ultimate levels due to aerodynamic smoothness or global stiffness requirements. The fuselage skin is often restricted from buckling up to a relatively low load level to control out-of-plane deformations at normal service conditions and to limit these deformations and the loss of effective in-plane stiffnesses at higher postbuckling load levels. Similar requirements exist for structural elements in many other types of applications. Design concepts that lead to increased plate-buckling loads (or strains) can directly lower the structural cost and/or weight by 1) allowing supporting substructure to be more widely spaced, thus reducing substructure, fastener, and attachment part count, 2) allowing the use of simpler substructure offering little rotational restraint to the surface structural elements, or 3) reducing the total structural weight by permitting design to a higher strain level governed by the material strength or damage tolerance rather than to lower values governed by buckling.

This study quantifies the improvements that can be achieved in compression buckling loads of rectangular composite plates by using a simple stiffness-tailoring concept. It provides guidelines that designers can use to achieve the improvement. Here stiffness tailoring is defined as the precise placement of lamina with various orientations through the thickness and across the planform of the laminated plate. The local membrane and bending stiffnesses of the tailored plate become nonuniform over the plate. When the loading is applied to the plate in the form of uniform imposed end displacements, stiffness tailoring brings about a redistribution of the in-plane loads that can directly benefit buckling response. At the same time, the distribution and magnitudes of the bending and twisting stiffnesses can also be modified to further improve the buckling load. The properly tailored design will be a function of the type of loading, the plate geometry, the boundary conditions, the relative material properties, and the basic configuration of the tailoring concept. Bulson[1] shows that the type of tailoring considered herein can provide a 42% improvement in the buckling load of isotropic plates loaded in pure compression. Much larger improvements will be shown when stiffness tailoring is applied to laminated composite plates.

Tailoring Approach

The tailoring concept evaluated herein involves redistribution of only 0-deg (x direction) material across the plate width and through the plate thickness, i.e., uniaxial tailoring. Preliminary studies showed that redistribution of all 0-deg material from the center region of the loaded ends toward the unloaded, supported edges tended to increase the x-direction buckling load N_x. Here N_x is the average load per unit width across the plate. Conversely, redistribution of the same material away from the edges toward the central region greatly reduced the buckling load. A planview and two cross-sectional views of this tailoring concept are shown in Fig. 1. The plate shown is a typical surface element supported by substructural elements (not shown). The substructure is assumed to prevent out-of-plane deformation on the perimeter of the plate.

For simplicity, and to minimize the increase in manufacturing complexity due to tailoring, only regions of two different uniform thickness laminate configurations are considered. This concept leaves all lamina continuous in their fiber directions and therefore creates no stress concentrations due to fiber dropoff in the fiber direction. Such plate geometries can easily be achieved with automated tape or tow placement machines, by pultrusion, by resin transfer molding (RTM) of stitched or woven preforms, or by conventional hand layup techniques. The effect of the small transition region between thick and thin regions on buckling is assumed to be small and these transition regions are not modeled. Local effects in these regions may be important in either local prebuckling or postbuckling stress distribution and failure prediction. This topic, although important, is not the focus of the present study.

Analytical Model

Buckling loads are computed for the tailored plates using the ABAQUS finite element code.[2] The elements chosen for the analysis are the eight-node quadrilateral anisotropic shell elements. The element stiffnesses are computed by numerically integrating the orthotropic lamina properties through the thickness using three integration points per layer. The elements are based on a first-order transverse shear-deformation theory that accounts for the effects of different lamina transverse shear stiffnesses in the longitudinal and lateral direc-

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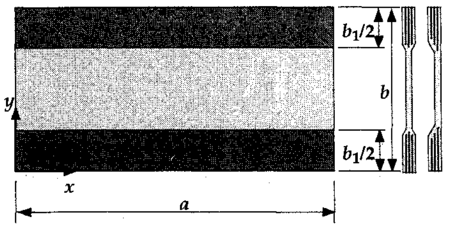

Fig. 1 Typical geometry for uniaxial stiffness tailoring.

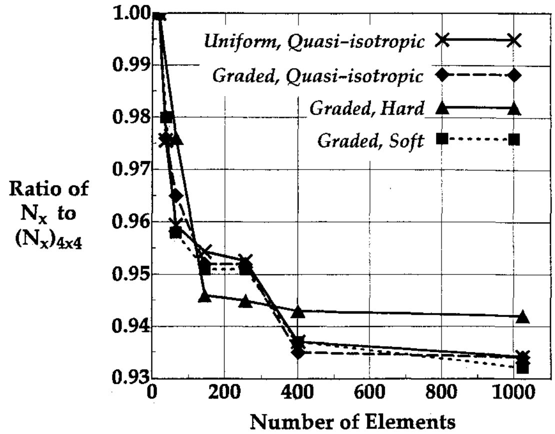

Fig. 2 Convergence of buckling load for simply supported, highly tailored ($\bar{b} = 0.25$), square plates.

tions. The relatively low transverse shear stiffness of composite materials can significantly affect the buckling loads of thick composite plates. This effect of transverse shear deformation becomes increasingly negative in regions where the plate thickness is increased due to tailoring. Transverse shear deformation is, therefore, included in the evaluation of tailoring so that the benefits of tailoring are not overestimated.

Material Properties

Material properties representative of an intermediate modulus fiber, toughened resin composite (IM7/8551-7) are used. Since all results are presented in relative or dimensionless form, only the relative properties of the material are of interest. The following properties were taken from Dow and Smith[3]:

$$E_{11}/E_{22} = 13.90, \qquad G_{12}/E_{22} = 0.48, \qquad \nu_{12} = 0.33$$

Values for G_{13} and ν_{13} are assumed equal to G_{12} and ν_{12}, respectively. A value for G_{23} was not available from tests and was therefore calculated from the micromechanics analysis of Tsai and Hahn[4] using the fiber volume fraction from Dow and Smith[3] and manufacturer's data for the shear stiffnesses of the matrix and the fiber. The resulting relative transverse shear stiffness is $G_{23}/E_{22} = 0.305$.

Model Refinement

Grid-refinement studies were first performed on isotropic, simply supported, square plates. Convergence to within 0.05% of the theoretical buckling load was achieved for plates with a 4×4 element mesh pattern. Since the elements in tailored plates will not necessarily be square or equally sized, various combinations of wide and narrow elements at the sides, ends, and center of the plate were evaluated while holding the number of elements at 16. In all cases, the buckling load varied by less than 0.4% from the theoretical value.

Similar grid-refinement studies were performed on tailored plates. In general, the number of elements required to reach a given degree of convergence increased with the degree of tailoring. A highly tailored plate is one in which the relocated material is concentrated in a narrow band at each side of the plate. A typical convergence study for such highly tailored plates with $\bar{b} = b_1/b = 0.25$ is shown in Fig. 2. Here the buckling loads are normalized with respect to values obtained with a 4×4 mesh. The results for a tailored, quasi-isotropic plate with a uniform grid may be compared to those for a graded grid in which smaller elements were used in the transition region between the central and edge regions. Element size grading produced only very slightly lower buckling predictions compared to models with a uniform mesh. The effect of the average membrane stiffness of the plate on convergence is also shown to be small, with the hard (increased axial stiffness) plates converging slightly faster than the quasi-isotropic and the soft (decreased axial stiffness) plates. Similar results generated for clamped edge supports lead to the same general conclusions but with slightly higher errors for models with the same number of elements compared to the results in Fig. 2.

Based on these studies and the fact that tailoring produces rather large (on the order of 50–200%) improvements in the buckling loads, it was decided to use models with uniform meshes ranging from 8×8 to 20×20, increasing in mesh density as the width ratio $\bar{b}$ decreased. The model refinement was chosen so as to limit errors in the values of the buckling loads to about 1–2% for the simply supported plates and to about 2–4% for plates with clamped edges.

Parametric Study

The design parameters considered include the width ratio $\bar{b} = b_1/b$, the average width-to-thickness ratio, the laminate stacking sequences in the thick and thin regions, the average in-plane stiffness, the boundary conditions, the plate aspect ratio, and whether the plate is flat on one surface or has a flat, continuous midplane as shown in the two cross-sectional views of Fig. 1. Unless noted, the discussion refers to plates with flat, continuous midplanes.

In all cases, the benefits of tailoring are shown in terms of the effects of the design parameters on the compressive buckling load ratio $\bar{N}_x$, defined as the average x-direction buckling load for the tailored plate divided by the buckling load of a corresponding uniform plate. The total in-plane stiffness in the major loading direction is always held constant in an integral sense over the plate width. Therefore, the improvements shown are not simply due to a change in total plate membrane stiffness that would require a corresponding change in the stiffnesses of other structural elements such as stiffeners, spar beam caps or webs, ribs, or other support structure. Likewise, the weight of the plate is not changed due to tailoring. With this approach, the benefits of tailoring are pure and are not achieved at the expense of structural changes not directly associated with the plate or an increase in the amount of material used in the plate or elsewhere in the structure.

Width Ratio $\bar{b}$

The geometric ratio of widths $\bar{b} = b_1/b$ is considered to be the most important design parameter in this study. It has a significant effect on the plate-buckling load and is easily controlled by the designer. The effect of the width ratio is therefore shown in conjunction with various other design parameters. The width ratio ranges from a value of 1.0 for uniform plates to small values in the range of 0.1–0.2 which correspond to narrow, thick, stiff regions near the unloaded edges. Ratios of edge thickness to the equivalent uniform thickness are plotted in Fig. 3 as a function of $\bar{b}$ for tailored quasi-isotropic plates and for a soft and hard laminate to be defined later.

Since the total average, or smeared, in-plane axial stiffness is held constant and equal to that of a uniform thickness plate in all cases, the width ratio controls the relocation of in-plane axial load across the width of the plate by changing the relative in-plane stiffness values. For a fixed sequence of lamina orientations through the thickness, the width ratio also controls the relative bending and twisting stiffness values of the thick and thin regions.

The smallest practical values of $\bar{b}$ should provide the most beneficial in-plane load distribution with the maximum portion of the total load located near the straight, supported edges. However, this same minimum value of $\bar{b}$ does not maximize the benefits of bending and twisting stiffness redistribution since the high stiffness values may be inefficiently concentrated too close to the edges. A tradeoff between these two effects of tailoring is expected to yield an optimum value of $\bar{b}$ between 1.0 and some minimum value close to zero. This expectation will be demonstrated in the following discussion. The points circled in Fig. 3 correspond to the geometry of tailored plates that correspond to these optimum values of $\bar{b}$. It can be seen that the optimum edge thickness ratios are under 1.75 in all cases, indicating that no extreme thickness variations are called for by this approach to tailoring.

Consider the effects of tailoring a quasi-isotropic plate with simply supported edges as shown in Fig. 4. It is clear that a reduction in $\bar{b}$ from 1.0 to 0.25 is beneficial, but that a further reduction below 0.25 has a negative effect. The corresponding maximum buckling load ratio is 2.38. This indicates that tailoring can increase the buckling load, or strain since the axial stiffness is held constant, by a factor of 2.38, or by 138%, compared to the uniform plate. This particular case is for a square, simply supported, thin plate. The reference uniform plate is a thin quasi-isotropic laminate with $[\pm 45/0/90]_s$ layup. The corresponding total width-to-thickness ratio is 150. The results shown represent cases with a $[\pm 45/90]_s$ thin central region and a $[\pm 45/0_n/90]_s$ thicker edge region. The value of n, defined simply as $1/\bar{b}$, was assumed to take integer and

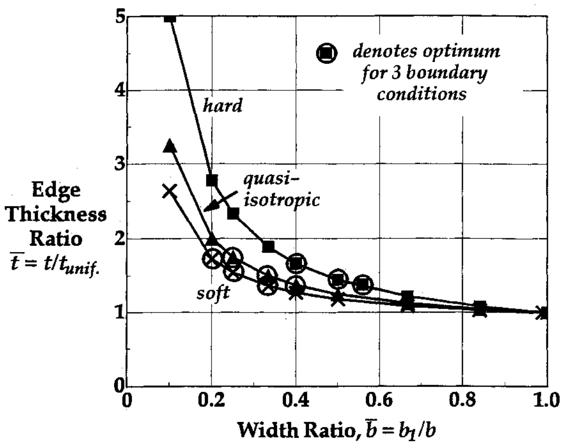

Fig. 3 Thickness ratios of the edge region of tailored plates relative to an equivalent uniform plate.

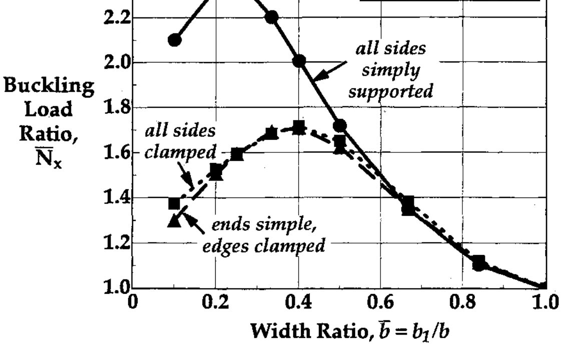

Fig. 4 Effects of tailoring on quasi-isotropic, thin, square plates.

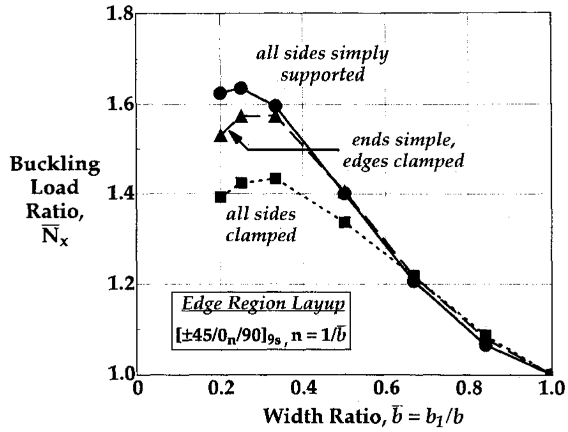

Fig. 5 Effects of tailoring on quasi-isotropic, thick, square plates.

noninteger values in developing the data. Integer values are attainable only for $\bar{b}$ = 1.0, 0.5, 0.333, 0.25, 0.20, etc. Intermediate values of $\bar{b}$ would require noninteger values of n. These could be achieved by using different ply thicknesses or by tow placement at specific areal weights.

Boundary Conditions and Average Width-to-Thickness Ratio

The results just discussed are for plates simply supported on all four sides. Data for square plates with all sides clamped, and with simply supported loaded ends and clamped unloaded edges are also presented in Fig. 4 for the thin plate. The normalizing buckling load in each case is that of the eight-ply uniform plate with the boundary conditions corresponding to that particular case. Therefore, as $\bar{b}$ approaches 1.0, all of the buckling ratios approach 1.0. The uniform plate-buckling loads with all sides clamped and with only the unloaded edges clamped are 2.10 and 1.73 times higher, respectively, than for the uniform simply supported case. These relative values are slightly different than those for classical isotropic plates due to the large difference in longitudinal and lateral bending stiffnesses and due to the bending-twisting coupling terms present in these thin plates.

Figure 4 shows that the width ratio producing the maximum increase in buckling load increases slightly as the edge restraint increases. However, the optimum width ratio for all cases remains in the range 0.25 to 0.4. It can also be seen that the benefit of tailoring is not as great for the plates with clamped edges as for the simply supported case. Tailoring produces only about 71% improvement with all sides clamped or with only the unloaded sides clamped. On the other hand, if it is determined that a larger than optimum width ratio is desired for practical reasons, say $\bar{b}$ = 0.667, the clamped edge conditions actually offer slightly more benefit due to tailoring than can be obtained with the simply supported conditions at the same nonoptimum width ratio.

Similar data are shown for the thick plates in Fig. 5. Here the reference uniform plate configuration is $[\pm 45/0/90]_{9s}$. The average width-to-thickness ratio is 16.67. The thin, central region laminate is $[\pm 45/90]_{9s}$ whereas the thicker edge regions are $[\pm 45/0_n/90]_{9s}$. The optimum width ratio ranges between 0.25 and 0.333 for all cases in Fig. 5, increasing only slightly as edge restraint increases. The corresponding maximum increases in buckling loads are about 43% with all sides clamped and 58% with the loaded ends simply supported and the unloaded edges clamped. These improvements compare to the 63% value achieved for the simply supported edge conditions.

Effects of Transverse Shear and Ply Grouping

In all cases, the thick plates offer less benefit due to tailoring than can be achieved with thin plates. There are two factors, namely transverse shear deformation and ply group-

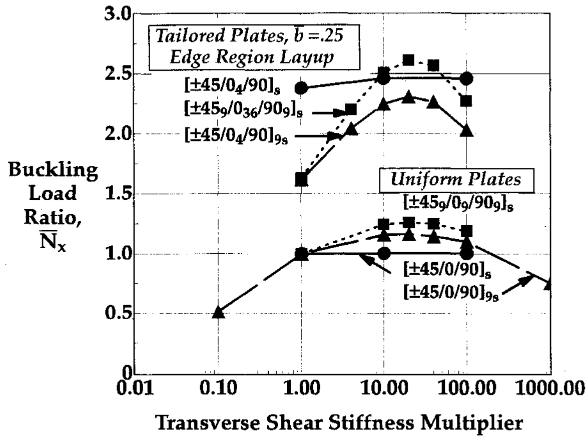

Fig. 6 Effects of transverse shear stiffness and ply grouping on simply supported, quasi-isotropic, thin and thick, square plates.

ing, that contribute to this difference in behavior. The data shown in Fig. 6 for simply supported, quasi-isotropic plates will help explain the effect of these factors. First consider the effects of transverse shear deformation. Buckling load ratios for uniform and tailored plates, both thin and thick, are shown as a function of transverse shear stiffness which was changed by multiplying the actual shear stiffnesses by a multiplier K_{ts}. Buckling loads for the uniform plates with $K_{ts} = 1.0$ are used to normalize the data. Consider the thin $[\pm 45/0/90]_s$ plate and its tailored counterpart. A change in K_{ts} from 1 to 100 produces essentially no change in the uniform thin plate-buckling load as expected. However, the buckling load for the tailored thin plate, with $\bar{b} = 0.25$, is slightly lower (about 3%) when transverse shear is considered than when it has been eliminated ($K_{ts} > 10$). This shows that transverse shear effects are greater for tailored plates, even thin ones, than for uniform plates.

Now consider the results for the thick $[\pm 45/0/90]_{9s}$ plate and its tailored counterpart. First it should be noted that values of K_{ts} above 20 should not be considered since they unrealistically cause a decrease in the predicted buckling loads. As a point of reference, an isotropic material with Poisson's ratio of 0.3 and with shear modulus equal to in-plane shear modulus of the composite laminates considered here would have a transverse shear modulus only 4 to 10 times the transverse shear moduli of the composite materials. Thus this restriction is not of practical concern. Considering that with $K_{ts} = 20$ the effects of transverse shear deformation have been eliminated, the transverse shear effect amounts to a 15% reduction for the uniform thick plates and 30% reduction for the tailored thick plates. Looked at another way, if transverse shear deformation were not a factor, tailoring the thick plates could produce a 131% increase in the buckling load, only slightly less than the 146% increase possible with tailored thin plates.

The effect of ply grouping can also be evaluated from the data in Fig. 6. In obtaining the thick-plate results just discussed, the ply orientations were uniformly distributed through the thickness of the uniform $[\pm 45/0/90]_{9s}$ laminate an its tailored counterpart as shown. By doing so, one loses some of the benefit of having the ±45-deg layers concentrated near the outer surfaces of the laminate. This can be seen from the results shown for the $[\pm 45_9/0_9/90_9]_s$ laminate. This highly efficient, but otherwise impractical, laminate is quite similar in its degree of ply grouping to the thin $[\pm 45/0/90]_s$ laminate. It is evident that the effect of transverse shear deformation is increased with increased ply grouping in the thick laminate. However, if transverse shear deformation were not a factor, the benefit of tailoring the thick, highly grouped laminate (an increase in $\bar{N}_x$ of 160%) would even exceed the 146% increase achieved with the tailored thin laminate. All data for thick plates shown subsequently were obtained considering transverse shear deformation and requiring plies with different orientations to be uniformly distributed through the laminates. As such, the improvements shown in the buckling load ratios can be thought of as lower bounds to the tailoring benefits that can be expected for thick plates.

Laminate Stacking Sequence

Results were also obtained by defining alternate stacking sequences in the thickened regions for simply supported, square, thin, and thick plates. The reference edge region laminates are $[\pm 45/0_n/90]_s$ and $[\pm 45/0_n/90]_{9s}$ for the thin and thick plates, respectively. Alternate stackings were achieved by interchanging the 90-deg material with various layers of the 0-deg material, starting at the outermost layer, e.g., $[\pm 45/90/0_n]_s$, $[\pm 45/0/90/0_{n-1}]_s$, etc. In these cases, the ±45-deg layers remain at the outside of the laminate or the repeating group of lamina. The data showed less than a 2% reduction in buckling load compared to the reference case for the thin plates and virtually no effect of these alternates on the thick plates.

Results are shown in Fig. 7 for a second set of alternate stacking sequences. Here the ±45-deg material has been interchanged with various layers of the 0-deg material in the thickened edge regions with $[90/\pm 45/0_n]_s$ layup. The buckling load used to normalize the data remains that of the reference $[\pm 45/0/90]_s$ uniform thickness laminate. The effect of moving the ±45-deg material away from the outer surfaces and closer to the midplane is extremely negative for these thin plates. The trend of decreasing buckling load is consistent as the ±45-deg layers are moved further toward the midplane or to the inside of the repeating group of lamina. Note that the values of the buckling load ratio $\bar{N}_x$ for these alternate stacking sequences approach values less than 1.0 as $\bar{b}$ approaches 1.0. This simply reinforces what numerous previous studies on uniform simply supported plate buckling have shown regarding the importance of placing the ±45-deg material as close as possible to the outer surfaces. If, however, some other design requirements dictate that the 90-deg layers be placed at or toward the outer surfaces, tailoring can still be effective in improving the buckling loads. In fact, the percentage increases, compared to uniform plates with the same $[90/\pm 45/0]_s$ or $[90/0/\pm 45]_s$ stacking sequences, are still substantial, i.e., well over 100%.

Tailoring thick plates, considering the same type of stacking sequence alterations just discussed for the thin plates, produces buckling load ratios that were less than 10% lower than those shown in Fig. 5. With thick plates, alternate stacking sequences that have major concentrations of adjacent lamina with the same orientation are not considered to be practical due to their tendency toward matrix cracking and higher interlaminar stresses. Such alternates were not extensively evalu-

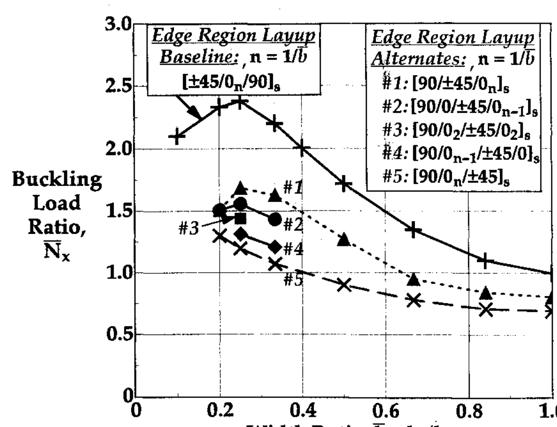

Fig. 7 Effects of tailoring and stacking sequence on quasi-isotropic, thin, simply supported square plates.

ated, although concentrating all ± 45-deg lamina at the outer surfaces can increase the benefit of tailoring as discussed in the preceding section. Alternates with 0-deg material on the outer surfaces are also not practical for longitudinally dominant loading due to the tendency for the outer load carrying plies to delaminate and were therefore not evaluated. If stitched or three-dimensionally woven preforms are used, either of these classes of alternates may be more reasonable and should be evaluated for the appropriate fabrication methods.

The effects of practical stacking sequence alterations are very minor on the thick plates and significant, but predictable, on the thin plates. Tailoring benefits for the thin plates are greater when stacking sequences that increase the buckling loads of uniform plates are used.

Aspect Ratio

Models of long plates with aspect ratio a/b equal to 5 were created to study the effect of length on the effectiveness of tailoring. Figure 8 presents data obtained from these models of thin plates for the three sets of boundary conditions discussed in the preceding section. All of the trends observed for the square plates hold true for the long plates with the exception, of course, that clamping the loaded ends of the long plate has negligible effect on the buckling load. Comparison with Fig. 4 clearly shows the strong similarity between the square and long plates. The long, simply supported plate shows essentially the same improvement in buckling load as the square plate. On the other hand, the long plates with clamped edges show slightly greater improvement due to tailoring compared to the square plates. This larger improvement is due primarily to the lower uniform plate-buckling load for the long plates compared to the square plates with clamped edges. Since these

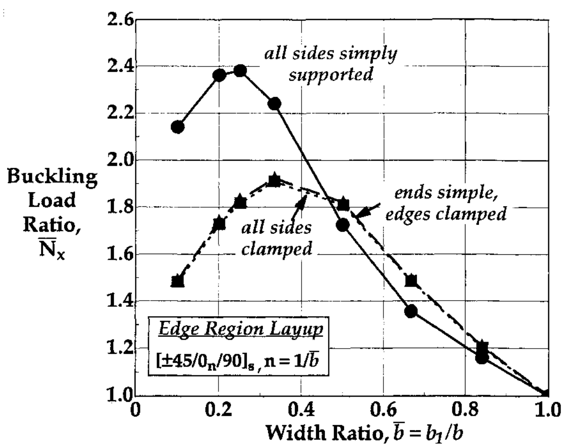

Fig. 8 Effects of tailoring on quasi-isotropic, thin, long plates.

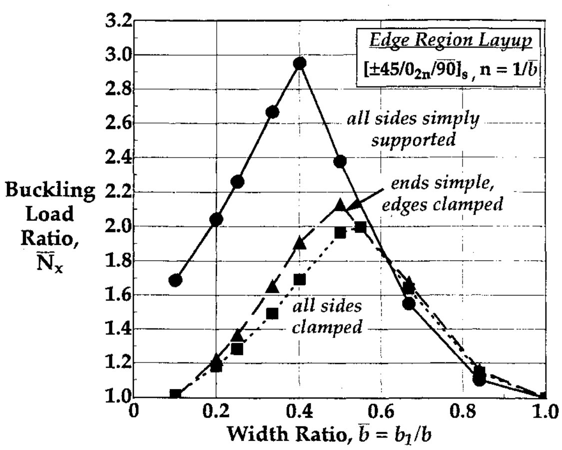

Fig. 9 Effects of tailoring on hard, thin, square plates.

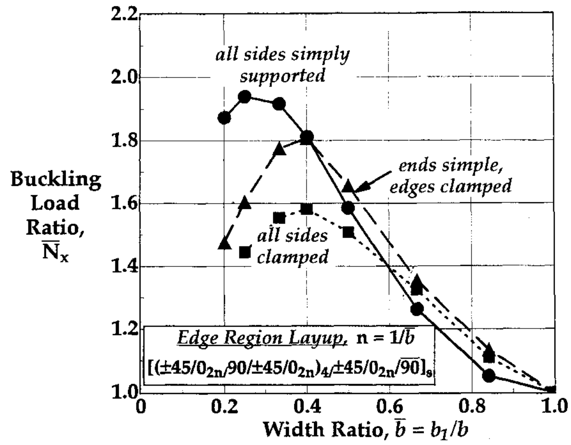

Fig. 10 Effects of tailoring on hard, thick, square plates.

differences between the square and long plates are rather small, data are not presented for intermediate aspect ratios or for long, thick plates.

Hard and Soft Laminates

Plates that are quasi-isotropic when uniform and smeared quasi-isotropic when tailored have been the focus of the preceding evaluations. Since some global structural configurations benefit from use of surface laminates with increased longitudinal stiffness (hard laminates), and since damage tolerance requirements sometimes dictate use of reduced stiffness (soft) plates on damage-prone exterior surfaces, typical hard and soft laminates were also evaluated for the effects of tailoring. The hard laminates used here, $[\pm 45/0_{2n}/\overline{90}]_s$ and $[(\pm 45/0_{2n}/90/\pm 45/0_{2n})_4/\pm 45/0_{2n}/\overline{90}]_s$, where $n = 1/\bar{b}$ as before, have 44.45% 0-deg and ± 45-deg fibers, and 11.1% 90-deg fibers. These laminates could represent a cover laminate in a multi-spar wing configuration in which the cover provides most of the wing bending stiffness. The soft laminates, $[\pm 45_s/0_n/\overline{90}]_s$ and $[(\pm 45_s/0_n/90/\pm 45_s/0_n)_4/\pm 45_s/0_n/\overline{90}]_s$, have 18% 0-deg, 73% ± 45-deg, and 9% 90-deg fibers. These laminates could be used for a stringer-stiffened multi-rib wing skin cover which is required to provide torsional strength and stiffness while providing maximum damage tolerance for wing bending loads. Comparison of the results for hard and soft plates to those for quasi-isotropic plates indicates similar trends but quantitative differences.

Compared to the quasi-isotropic laminate, the hard, thin laminate shows larger increases in buckling loads as illustrated in Fig. 9. Since hard laminates have more 0-deg material to redistribute, they should be expected to yield larger tailoring benefits than quasi-isotropic laminates. Peak percentage increases in buckling loads are now 195, 113, and 100 for the simply supported, two sides clamped, and all sides clamped conditions, respectively. These increases are substantially larger than those obtained for quasi-isotropic laminates. The optimum width ratios are also larger and a narrower range exists around these optimum values where large benefits may be obtained.

Analyses were also performed for long, hard, thin plates with an aspect ratio of 5. Here again the effects of clamped loaded ends are negligible. The peak increases in the buckling loads ratios are 205 and 144% for the simply supported case and for the clamped edge cases, respectively. Substantially more tailoring benefit exists for the hard laminates with clamped edges when the plate is long rather than square.

Data for hard, thick, square plates are shown in Fig. 10. Again the trends closely resemble those observed for quasi-isotropic thick plates with some quantitative differences. The optimum width ratios are now slightly more affected by edge restraint and range from 0.25 to 0.40. Increases in buckling load due to tailoring are, however, significantly larger than for

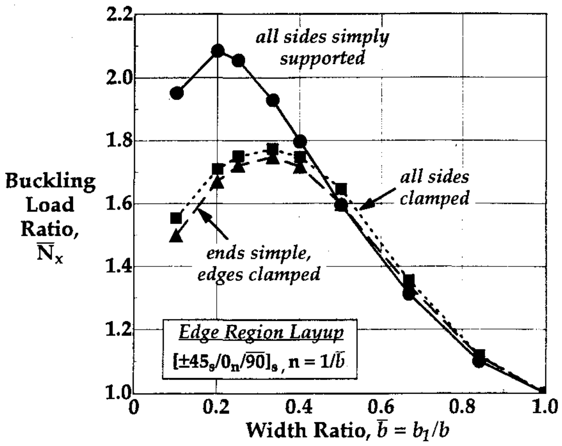

Fig. 11 Effects of tailoring on soft, thin, square plates.

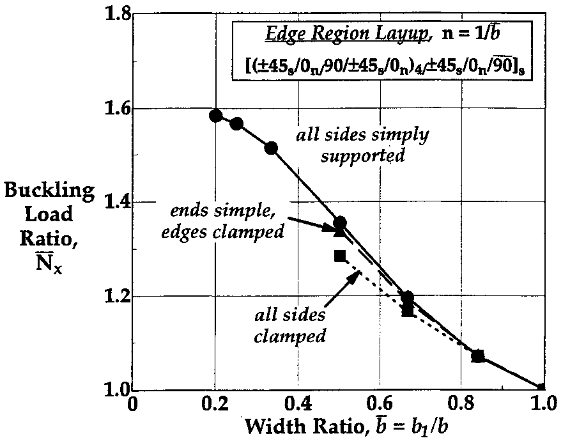

Fig. 12 Effects of tailoring on soft, thick, square plates.

thick quasi-isotropic plates. Peak increases observed are now 94, 81, and 58% for the simply supported, two sides clamped, and all edges clamped conditions, respectively.

The soft, thin, square plate results, shown in Fig. 11, indicate smaller optimum width ratios and somewhat smaller increases in buckling loads, ranging from 75 to 108%, compared to the quasi-isotropic and hard laminates. The soft laminates, however, show a wider range around the optimum width ratio where substantial tailoring benefits may be obtained.

Similar results for soft laminate, thick, square plates are shown in Fig. 12. Data are not shown for $b_l < 0.5$ for the clamped-edge cases since the model failed to predict reasonable results for these cases. The data for the simply supported case indicate that the optimum width ratio is less than 0.20, the smallest value considered practical for thick plates. The maximum increase in buckling load ratio is about 60% for the simply supported case.

Tailored Plates with One Surface Flat

When one surface of the plate is required to be flat, the midplane of the plate becomes discontinuous as it transitions between the thick and thin across the width of the plate. Although this discontinuity is expected to have an effect on the response of the plate to lateral in-plane loading, and perhaps to in-plane shear loading as well, its effect on the response to longitudinal compressive loading is not as obvious. Finite element models were created with an offset present between the midplanes of the thin central regions and thickened edge regions of a representative tailored plate. Constraints were placed on the relative deformations of elements in two regions that are consistent with the first-order shear-deformation elements used.

The thin, quasi-isotropic plate with a width ratio $\bar{b} = 0.50$ was used for this investigation. The boundary conditions on the loaded ends are of concern here. The uniform end deformation was imposed along a straight line defined as the axial stiffness-weighted "neutral axis" for the tailored plate. If rotations on the loaded ends are restrained, the midplane discontinuity causes no difficulty in the analysis and the buckling load actually increases about 3–4% compared to a plate with the same clamped loaded ends but with a flat, continuous midplane.

If rotations on the loaded ends are permitted, application of the end displacement produces rotations (and associated plate bending) in one direction in the thickened edge regions and in the opposite direction in the thin central region. The rotations are constrained to become equal at the intersection of the two regions. Although the total end moment applied to the plate is in fact zero as required by a "simply supported" boundary condition, local moments of opposite signs are applied along the thick and thin regions. These moments and the associated bending cause the response of the plate to be nonlinear from the initiation of loading. Bifurcation buckling does not physically exist for this class of structure and the existence of limit point buckling response must be determined from a general nonlinear analysis. However, if this nonlinearity is ignored, computation of the linear elastic bifurcation buckling load yields a value 1.05 times the buckling load of the corresponding plate with a continuous midplane.

To eliminate this nonlinearity introduced by attempting to impose a simply supported end condition, a different approach was taken. Three rectangular plate models with aspect ratios 3, 5, and 7 were constructed. Since the effects of clamped loaded ends have been shown to become negligible for long plates (a/b = 5), the loaded ends of the plates were clamped. The unloaded edges were simply supported. In each case, transverse deformations were prevented along two lateral lines defining a square region in the center of each rectan-

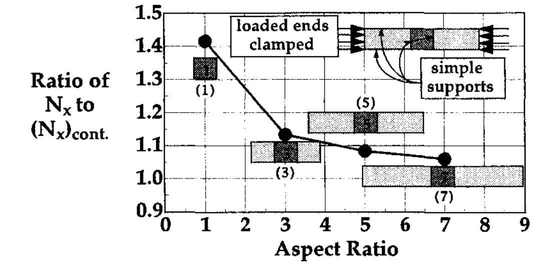

Fig. 13 Buckling load for an equivalent "simply supported, square," tailored ($\bar{b}$ = 0.5) plate with one surface flat compared to a simply supported, square plate with a flat, continuous midplane.

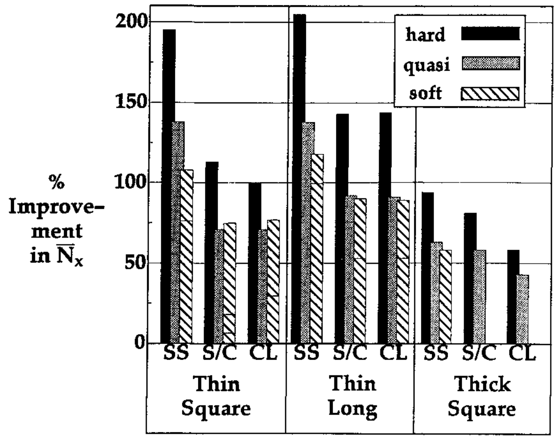

Fig. 14 Summary of the maximum improvements in the compressive buckling loads of square and rectangular plates due to stiffness tailoring.

gular plate. In this way, an attempt was made to create an "equivalent simply supported square" plate in the center of each rectangular plate and to eliminate the effect of the rotational restraint at the loaded ends. Buckling load ratios for $\bar{b} = 0.50$, normalized by the buckling load of the simply supported square plate with continuous midplane, are plotted vs aspect ratio in Fig. 13. The buckling load ratio appears to be converging to a value very close to the 1.05 factor obtained for the square, simply supported plate when the nonlinear response was ignored. These results indicate that, with respect to linear bifurcation buckling, the simply supported square plate model and the "equivalent simply supported square" plate model predict the same buckling load. This agreement and the fact that the discontinuity in the midplane slightly increases the buckling load, just as it does for the clamped square plate, give some confidence that tailoring with one surface flat should be considered a reasonable alternative to tailoring with a flat, continuous midplane. Further investigation by means of a nonlinear analysis is required to verify this conclusion.

Summary and Conclusions

A simple stiffness-tailoring concept has been shown to be an effective way to increase the buckling load or strain in composite plates loaded in compression with no sacrifice in average in-plane stiffness or weight. A summary of the results of this study is shown in the chart in Fig. 14. The boundary conditions are denoted SS for all sides simply supported, S/C for simply supported loaded ends and clamped unloaded edges, and CL for all sides clamped. The percentage improvements shown are relative to a uniform plate with the same boundary conditions. The figure shows that tailoring is especially advantageous when the plate is relatively thin, when the unloaded edges are simply supported, and when the average in-plane axial stiffness of the laminate is relatively high. Laminate stacking sequence has only a small effect on the benefits of the tailoring concept considered. Plate length effects are also small, except for hard plates with clamped unloaded edges.

Some specific conclusions reached in this research are as follows:

1) A simple redistribution of 0-deg material from a central region toward supported edge regions can produce increases in buckling loads, or buckling strains, in the range of 200% compared to uniform plates.

2) Optimum width ratios $\bar{b}$ generally range from 0.25 to 0.5, increasing with increased edge rotational restraint and the percentage of 0-deg lamina in the plate.

3) The maximum percentage increase in buckling load that can be achieved increases as the average width-to-thickness ratio of the plate increases. Transverse shear-deformation affects the buckling loads of tailored plates more than uniform plates. Efficient ply grouping, if practical, can increase the benefits of tailoring.

4) The trends of tailoring benefits are unchanged for minor alterations in the stacking sequence of a given laminate. However, magnitudes of buckling loads of tailored simply supported plates, as well as the percentage increase due to tailoring, always decrease as ± 45-deg layers are moved from outer to inner locations just as is the case for uniform simply supported plates.

5) Tailoring provides greater maximum percentage increases in buckling load when the unloaded edges are simply supported rather than clamped. If, however, nonoptimum width ratios are required for some reason, tailoring can be more effective with clamped edges than with simply supported edges in some cases.

6) The length-to-width aspect ratio has little impact on the effects of tailoring. However, for hard plates with clamped unloaded edge conditions, long plates show somewhat more benefit due to tailoring than do square plates.

7) Hard laminates show larger positive effects of tailoring than do quasi-isotropic laminates.

8) Soft laminates show slightly smaller, but still very significant, increases in buckling load due to tailoring compared to quasi-isotropic simply supported plates, and approximately the same degree of improvement as quasi-isotropic plates with clamped unloaded edges.

9) Initial indications are that the benefits of tailoring are not degraded for plates that are required to have one surface flat and therefore have a midplane discontinuity, rather, the benefits are slightly greater than for plates with flat, continuous midplanes.

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